

MATHEMATICS(HONOURS)

SEMESTER-I C:1-CALCULUS-I

(Total Marks: 100)

Part-I(Marks: 70)

4 Lectures, 1 Tutorial (per week)

Unit-I

Hyperbolic functions, higher order derivatives, Leibnitz rule and its applications to problems of the type $e^{ax+b} \sin x$; $e^{ax+b} \cos x$; $(ax + b)^n \sin x$; $(ax + b)^n \cos x$; concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule, applications in business, economics and life sciences.

Unit-II

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x dx$; $\int \cos^n x dx$; $\int \tan^n x dx$; $\int \sec^n x dx$; $\int (\log x)^n dx$; $\int \sin^n x \cos^n x dx$; volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parameterizing a curve, arc length, arc length of parametric curves, area of surface of revolution.

Unit-III

Techniques of sketching conics, reflection properties of conics, rotation of axes and second degree equations, classification into conics using the discriminant, polar equations of conics. Sphere, Cone, Cylinder, Conicoids.

Unit-IV

Vector triple product, Introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration.

C:2-ALGEBRA-I

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Polar representation of complex numbers, n-th roots of unity, De Moivre's theorem for rational indices and its applications.

Unit-II

Equivalence relations, Basic Terminology, Functions, Inverse and composition of functions, One-to-One correspondence and cardinality of a set, Division algorithm, Divisibility and Euclidean algorithm, Prime numbers, Congruence relation between integers, Principles of Mathematical Induction, State-ment of Fundamental Theorem of Arithmetic.

Unit-III

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation $Ax = b$; solution sets of linear systems, applications of linear systems, linear independence.

Unit-IV

Introduction to linear transformations, Matrix of a linear transformation, Inverse of a matrix, Char-acterizations of invertible matrices. Subspaces of \mathbb{R}^n , Dimension of subspaces of \mathbb{R}^n and Rank of a matrix, Eigen values, Eigen Vectors and Characteristic equation of a matrix.

SEMESTER-II

C:3-REAL ANALYSIS (ANALYSIS-I)

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Review of Algebraic and Order Properties of \mathbb{R} ; Upper bound & Lower bound, Least upper bound (LUB), Greatest lower bound (GLB), LUB & GLB property of an ordered eld, Completeness of an ordered eld, Incompleteness of \mathbb{Q} , Supremum and In mum, Roots, Archimedean property, Rational & Irrational density theorems, Decimal representations of real numbers.

Unit-II

Idea of countable, uncountable sets and theorems relating to these sets, Sequences, Convergence & divergence of sequences, Limit of a sequence & Limit Theorems, Monotonic sequences, Weierstrass completeness principle, Nested Intervals, Cantor's completeness principle, Idea about higher order cardinals (restricted).

Unit-III

Subsequences, Bolzano Weierstrass theorem for sequences, Cluster points, Cauchy(Fundamental) sequence, Cauchys Convergence Criterion, Limit superior and Limit inferior, Convergence and diver-gence of in nite series, Series of positive terms, Tests of convergence.

Unit-IV

Absolute convergence, Rearrangement of terms of a series, Conditional convergence of a series, Open sets, Closed sets, Limit points, Closure, Interior and Boundary of sets. Bolzano Weierstrass theorem for sets.

C-:4-DIFFERENTIAL EQUATIONS

(Total Marks:100)

Part-I(Marks: 70)

4 Lectures, 1 Tutorial (per week)

Unit-I

Basic concepts of Differential equations and mathematical models. First order and first degree Ordinary differential equations(variables separable, homogeneous, exact, and linear). Applications of first order differential equations(Growth, Decay and Chemical Reactions, Heat flow, Oxygen debt, Economics). Equations of first order but of higher degree.

Unit-II

Second order linear equations(both homogeneous and non-homogeneous) with constant coefficients, second order equations with variable coefficients, variation of parameters, method of undetermined coefficients, Euler's equation, Second order differential equations with variable coefficients, Equations reducible to linear equations with constant coefficients.

Unit-III

Power series solutions of second order differential equations.

Unit-IV

Laplace transforms and its applications to solutions of differential equations.

SEMESTER-III

C-5: THEORY OF REAL FUNCTIONS (ANALYSIS-II)

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Limits of functions (approach), Sequential criterion for limits, Divergence criteria. Limit theorems, one-sided limits. Infinite limits and limit at infinity. Continuous functions, Sequential criterion for continuity, Algebra of continuous functions and theorems related to continuity of functions.

Unit-II

Discontinuity and kinds of discontinuity, Further properties of continuity, Uniform continuity, Differentiable functions, Left hand & Right hand derivatives, Algebra of differentiable functions, Carathéodory's theorem.

Unit-III

Mean value conditions, Global and local maximum & minimum, Rolle's theorem, Generalized mean value theorem, Cauchy mean value theorem, Lagrange's mean value theorem and their applications, Darboux's theorem, Indefinite forms, Higher order derivatives (Leibnitz theorem), Taylor's theorem and its applications to approximating functions by means of polynomials.

Unit-IV

Maxima and Minima, Taylor's theorem with different forms of remainder, Maclaurin's theorem, Deduction of Taylor's theorem from mean value theorem, Taylor's and Maclaurin's infinite series, Taylor's series and Maclaurin's series expansions of exponential and trigonometric functions, $\ln(1+x)$; $\frac{1}{1+(ax+b)}$ and $(1+x)^n$:

C-6: GROUP THEORY (ALGEBRA-II)

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), Elementary properties of groups.

Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a group, Product of two subgroups.

Unit-II

Properties of cyclic groups, Classification of subgroups of cyclic groups. Cycle notation for permutations, Properties of permutations, Even and Odd permutations, Alternating group, Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

Unit-III

External direct product of a finite number of groups, Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

Unit-IV

Group homomorphisms, properties of homomorphisms, Cayley's theorem, Properties of isomorphisms, First isomorphism theorem, Second and Third isomorphism theorems (Statements only).

C-7: PARTIAL DIFFERENTIAL EQUATIONS & SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

(Total Marks: 100)

Part-I(Marks: 70)

04 Lectures(per week)

Unit-I

Systems of Linear Differential Equations: Basic theory of linear systems, Trial solution method for linear system with constant coefficients, Simultaneous linear first order equations in three variables, Methods of solution, Pfaffian differential equations, methods of solutions of Pfaffian differential equations in three variables.

Unit-II

Formation of first order partial differential equations, Linear and non-linear partial differential equations of first order, Special types of first-order equations, Solutions of partial differential equations of first order satisfying given conditions.

Unit-III

Linear partial differential equations with constant coefficients, Equations reducible to linear partial differential equations with constant coefficients, Partial differential equations with variable coefficients, Some standard forms of variable coefficients.

Unit-IV

Laplace equation, Solution of Laplace equations by separation of variables, One-dimensional Wave equation, Solution of the Wave equation(method of separation of variables), Diffusion equation, Solution of one-dimensional diffusion equation, Method of separation of variables.

SEMESTER-IV

C-8: NUMERICAL METHODS

(Total Marks: 100)

Part-I(Marks: 70) 04
Lectures(per week)

Unit-I

Rate of convergence, Algorithms, Errors: Relative, Absolute, Round off, Truncation. Numerical solution of non-linear equations : Bisection method, Regular-Falsi method, Secant method, Newton-Raphson method, Fixed-point Iteration method, Newton-Raphson method for multiple roots, Aitken's $\frac{4}{2}$ process, Muller's method. Rate of convergence of these methods.

Unit-II

System of linear equations: Gaussian Elimination method, Gauss-Jordan method, Gauss Jacobi method, Gauss-Seidel method and their convergence analysis, .

Unit-III

Polynomial interpolation: Existence uniqueness of interpolating polynomials, Lagrange and Newtons divided difference interpolation, Error in interpolation, Central difference & averaging operators, Gauss-forward and backward difference interpolation, Simple numerical methods for derivatives, Interpolatory formulas.

Unit-IV

Numerical Integration: Some simple quadrature rules, Newton-Cotes rules, Trapezoidal rule, Simpson's rule, Simpson's $\frac{3}{8}$ -th rule, Compound quadrature rules, Compound midpoint rule, Compound Trapezoidal rule, Compound Simpson's rule, Gauss-Legendre 2-point & 3-point rules. Numerical solutions of Differential Equations: Eulers method. Runge-Kutta methods of orders two, three and four.

C-9: RIEMANN INTEGRATION & SERIES OF FUNCTIONS

(ANALYSIS-III)

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Riemann integration, Inequalities of upper and lower sums, Riemann conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums, Equivalence of two definitions, Riemann integrability of monotone and continuous functions, Properties of the Riemann integral, Definition and integrability of piecewise continuous and monotone functions, Fundamental theorems of Calculus.

Unit-II

Improper integrals; Series and Integrals, Absolute convergence of integrals, Convergence of Beta and Gamma functions.

Unit-III

Point-wise and Uniform convergence of sequence of functions, Cauchy's criterion & Weierstrass M-test for uniform convergence, Dedekind test, Uniform convergence and Continuity, Term by term integration of series, Term by term differentiation of series.

Unit-IV

Power series (Cauchy Hadamard Theorem), Radius of convergence, Differentiation and integration of power series, Abels Limit Theorem, Stirling's formula, More about Taylor's series, Weierstrass Approximation Theorem.

C-10: RING THEORY & LINEAR ALGEBRA (ALGEBRA-III)

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Definition and examples of rings, Properties of rings, Subrings, Integral domains and Fields, Characteristic of a ring, Ideal, Ideal generated by a subset of a ring, Factor rings, Operations on Ideals, Prime and Maximal ideals.

Unit-II

Ring homomorphisms, Properties of ring homomorphisms, Isomorphism Theorems I, II and III, Field of quotients.

Unit-III

Vector spaces, Subspaces, Algebra of subspaces, Quotient spaces, Linear combination of vectors, Linear span, Linear independence, Basis and Dimension, Dimension of subspaces.

Unit-IV

Linear transformations, Null space, Range, Rank and Nullity of a linear transformation, Matrix representation of a linear transformation, Algebra of linear transformations. Isomorphisms, Isomorphism theorems, Invertibility and Isomorphisms, Change of co-ordinate matrix.

SEMESTER-V

C-11: MULTIVARIATE CALCULUS (CALCULUS-II)

Total Marks: 100-(Theory:80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Functions of several variables, limit and continuity of functions of two variables, Partial differentiation, Tangent planes, Approximation and Differentiability, Chain rule for one and two independent parameters.

Unit-II

Directional derivatives and gradient, Maximal property of the gradient, Normal property of the gradient, Tangent planes and the normal lines, Extrema of functions of two variables, Method of Lagrange multipliers, Lagrange Multipliers, Constrained optimization problems, A geometrical interpretation.

Unit-III

Double integration over rectangular region and over non-rectangular region, Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions, Volume by triple integrals. cylindrical and spherical co-ordinates. Change of variables in double integrals and triple integrals.

Unit-IV

Definition of vector field, Divergence and Curl, Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem and path independence for line integrals.

Unit-V

Green's theorem, Area as a line integral, Alternative forms of Green's theorem, Normal derivatives, Surface integrals, Integrals over parametrically defined surfaces. Stokes theorem, The Divergence theorem.

C-12: PROBABILITY & STATISTICS

Total Marks:100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Sample space, Probability axioms, Independent events, Conditional probability & Bayes' theorem, Real random variables (discrete and continuous), Cumulative distribution function, Expectation of random variables, Some special expectations.

Unit-II

Multivariate distributions, Joint cumulative distribution functions, Joint probability distributions, Marginal & conditional distributions, Some probability distributions(Discrete case), Uniform distribution, Binomial distribution, Negative Binomial & Geometric distributions, Poisson distribution.

Unit-III

Some probability distributions(Continuous case), Uniform, Gamma, Exponential, Beta distributions, Normal distributions, Normal approximation to the Binomial distribution, Bivariate normal distribution.

Unit-IV

Distribution of two random variables, Expectation of function of two random variables, Moment generating functions, Conditional distributions & expectations, Correlation coefficient, Co-variance, Independent random variables, Linear regression for two variables.

Unit-V

Limit theorems, Markov's inequality, Chebyshev's inequality, Statement and interpretation of Weak and Strong law of large numbers, Central Limit theorem for independent and identically distributed random variables with finite variance, Markov Chains: Introduction, Chapman-Kolmogorov equations.

SEMESTER-VI

C-13: METRIC SPACES & COMPLEX ANALYSIS (ANALYSIS-IV)

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Metric spaces: Definition and examples, Open & Closed spheres, Neighborhoods, Interior points, Open set, Closed set, Boundary points, Limit points & isolated points, Closure of a set, Dense sets, Separable metric spaces, Sequences in metric spaces, Convergent sequences, Cauchy sequences, Complete metric spaces, Distance between sets & diameter of a set, Subspaces, Cantor's theorem.

Unit-II

Continuous functions: Definition & characterizations, Sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Connectedness, Connected subsets of \mathbb{R} ;

Separated sets, Disconnected sets, Contraction mappings, Banach Fixed point theorem.

Unit-III

Properties of complex numbers, Regions in the complex plane, Functions of complex variable, Map-pings, Limits & Continuity of complex functions, Derivatives, Differentiation formulas, Cauchy-Riemann equations, Sufficient conditions for differentiability, Polar Coordinates, Analytic functions, Examples of analytic functions.

Unit-IV

Exponential function, Logarithmic function, Trigonometric function, Derivatives of these functions, Definite integrals of functions, Contours, Contour integrals and its examples, Upper bounds for moduli of contour integrals, Theorems on antiderivatives, Cauchy-Goursat theorem (statement only), Cauchy integral formula, Its extension and consequences.

Unit-V

Liouville's theorem and the Fundamental theorem of Algebra, Convergence of sequences and series, Taylor series with examples, Laurent series (without proof) with examples, Absolute and uniform convergence of power series.

C-14: LINEAR PROGRAMMING

Total Marks: 100-(Theory: 80 Marks+Mid-Sem: 20 Marks) 5
Lectures, 1 Tutorial (per week)

Unit-I

Introduction to linear programming problems(LPP), Mathematical formulation of the LPP with illustrations, Graphical method, General Linear programming problems, Canonical & standard form of LPP.

Unit-II

Theory of Simplex method, Optimality and unboundedness, the Simplex algorithm, Simplex method in tableau format, Introduction to artificial variables, Two-phase method, Big-M method and their comparisons.

Unit-III

Duality in LPP: Introduction, General Primal-Dual pair, Formulation of the Dual problem, Primal-Dual relationships, Duality theorems, Complementary slackness theorem, Duality & Simplex method, Economic interpretation of the Duality.

Unit-IV

Transportation Problem(TP): LP formulation of TP, Existence of solution and Duality in TP, Solution of Transportation problems, North-West corner method, Least-Cost method and Vogel approximation method for determination of starting basic solution, Algorithm for solving transportation problem, Assignment problem and its mathematical formulation, Solution methods of Assignment problem, Special cases in Assignment problems.

Unit-V

Games and Strategies: Introduction, Formulation of two person zero sum games, solving two person zero sum games, Maximin-Minimax principle, Games without saddle points, Games with mixed strategies, Graphical solution procedure to $(2 \times n)$ and $(m \times 2)$ games.